An Expression for the S-parameters of Arbitrarily Oriented Microstrip Lines

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Abstract

An approximate expression for the coupled S-parameters of skew microstrip lines is derived. The expression is compared to the model for parallel coupled lines. An experiment to test the approximation for skew lines is described, and measured and predicted results are compared.

Introduction

Parasitic coupling is the unintended electromagnetic coupling between two circuit elements. This type of coupling can have deleterious effects on the circuit's performance, even though the absolute level of coupling may be quite small. [1] For example, the input stage of an amplifier may couple to the output stage in such a way that the feedback degrades the circuit's performance. While in principle full wave field simulators can calculate the parasitic coupling, in practice they are not a desirable solution because of the excessive computer resources required. An approximation that is less accurate than a full wave analysis, but much faster, can be used to pinpoint a parasitic coupling problem. Than a more accurate analysis can be employed.

For this purpose, an approximate expression for the S-parameters of weakly coupled skew microstrip lines is derived. It is then compared to a standard model for parallel coupled microstrip lines, and to experimental results for skew lines. These comparisons show the accuracy that can be expected.

Derivation of S-parameter Expression

The starting point in the derivation is the two dimensional EFIE [2] for the microstrip problem. Since the coupling is weak, second order coupling effects are ignored. The surface currents on a microstrip line are replaced by a line current at the center of the line. Consider two lines in isolation, although they are actually embedded in some circuit. Artificially place the currents for a quasi-TEM mode on one line, and calculate the modal currents impressed on the second line. Then normalize to get the S-parameters.

To compute the currents impressed on the second line, a one dimensional integral equation for the currents on an infinite microstrip line, due to an assumed incident electric field, is derived; then an approximate solution to this equation is found.

Start with the two dimensional integral equation [2].

$$\vec{E}_t^{(inc)}(\vec{\rho}) = j\omega \int dS' \, \vec{J}(\vec{\rho}')g_A(|\vec{\rho}-\vec{\rho}'|) + \nabla_t \int dS' \, q(\vec{\rho}')g_V(|\vec{\rho}-\vec{\rho}'|)$$

The left hand side is the tangential incident electric field and the right hand side is the negative of the tangential scattered electric field. $\vec{J}$ and q are the surface current and charge on the line, and $\vec{\rho}$ is a point on the line.

Assume a current of the form $\vec{J}(x',y') = \vec{a}_y f(x')I(y')$, where $I(y')$ is the *total* current on the line, and $f(x')$ gives the shape of the current in the transverse direction. Because transverse currents are ignored, the charge will have the same transverse variation or shape. Take $f(x)$ to be the static charge distribution on a microstrip line.

Now the two dimensional integral equation is multiplied by a weight function and integrated over the transverse coordinate. Since the assumed current and charge on the line are symmetric about the center of the line, the scattered field will be also. The weight function should therefore be symmetric. The shape function $f(x)$ is used; this has an advantage that will be pointed out later.

The desired one dimensional integral equation is:

$$j\omega \int_{-\infty}^{\infty} dy' I(y')k_A(|y-y'|) - \frac{1}{j\omega}\frac{\partial}{\partial y}\int_{-\infty}^{\infty} dy' \frac{\partial I(y')}{\partial y'} k_V(|y-y'|) = r(y) = \vec{a}_y \cdot \int_{-w/2}^{w/2} dx' f(x')\vec{E}^{inc}$$

The kernel k_A of this new integral equation is

$$k_A(y) = \int_{-w/2}^{w/2} dx \, f(x) \int_{-w/2}^{w/2} dx' \, f(x') g_A(\sqrt{y^2 + (x-x')^2});$$

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k_V is defined in a similar fashion.

This integral equation can be solved exactly. Take the Fourier transform of both sides of the equation, and apply the convolution theorem:

$$[-\omega^2 K_A(\beta) + \beta^2 K_V(\beta)]I(\beta) = j\omega R(\beta)$$

where β is the transform variable, and R, K_A and K_V are the Fourier transform of r, k_A and k_V respectively. The solution for the total current on the microstrip line is:

$$I(y) = \frac{1}{2\pi} \int_{-\infty}^{\infty} d\beta \, e^{-j\beta y} \frac{j\omega R(\beta)}{\beta^2 K_V(\beta) - \omega^2 K_A(\beta)}$$

It is probably not possible to find an analytic expression for $I(y)$. Therefore, try to find an asymptotic approximation for $|y| \to \infty$. Assume the poles in the integrand occur at $\beta_0 h \ll 1$. Because the charge distrubution was used as the weighting function above, $K_V(\pm\beta_0) \approx 1/C_e$, and $K_A(\pm\beta_0) \approx L_e$ [3] , where C_e and L_e are the capacitance and inductance per unit length of the line. This is the advantage mentioned above. Notice the assumption here that K_A and K_V vary *slowly* near $\beta = 0$. This is true because of the singularities in k_A and k_V, but a rigorous proof is not given. The poles are at $\beta = \pm\beta_0$, $\beta_0 = \omega\sqrt{L_e C_e}$. One finds:

$$I(y) \sim \frac{\omega C_e}{2\beta_0} R(\beta_0) e^{-j\beta_0 y} \qquad y \to \infty$$

$$I(y) \sim \frac{\omega C_e}{2\beta_0} R(-\beta_0) e^{j\beta_0 y} \qquad y \to -\infty$$

For a four port network made up of two stright microstrip line segments with each end terminated by the characteristic impedance of the line, define the following:

$\vec{r}_m$	center of line m
$\vec{u}_{m,i}$	unit vector along line m away from port i
$\vec{\rho}_m = \vec{r}_m + \vec{u}_{m,i}\zeta_m$	position on line m
β_m	wave number for line m
Z_m	characteristic impedance of line m
L_m	length of line m
$\vec{I}_{m,i} = \vec{u}_{m,i} e^{-j\beta_m \zeta_m}$	quasi-TEM line current
$a_{m,i} = \sqrt{Z_m} e^{\beta_m L_m/2}$	wave variable for port i on line m
$b_{m,i}$	wave variable for port i on line m
$S_{\substack{n,j\\m,i}}$	S-parameter

The coupled S-parameter is

$$S_{\substack{n,j\\m,i}} = \frac{b_{n,j}}{a_{m,i}} = -\frac{1}{2}\sqrt{Y_m Y_n} e^{-j(\beta_m L_m + \beta_n L_n)/2} R(-\beta_n).$$

The calculation of the coupling boils down to computing

$$R(-\beta_n) = \int_{-\infty}^{\infty} dy \, r(y) \; e^{j\beta y}.$$

Two observations are made: first $r(y)$ is assumed to be zero except on $[-L_n/2, L_n/2]$, by truncation, and second $r(y)$ can be written in terms of an incident $\vec{A}$ and ϕ.

$$r(\zeta_n) = \vec{u}_{n,j} \cdot \vec{E}^{inc} = -j\omega \vec{u}_{n,j} \cdot \vec{A} - \frac{\partial}{\partial \zeta_n}\phi$$

Finally,

$$\begin{aligned} R(-\beta_n) = {} & -j \int_{-L_m/2}^{L_m/2} d\zeta_m e^{-j\beta_m \zeta_m} \int_{-L_n/2}^{L_n/2} d\zeta_n e^{-j\beta_n \zeta_n} \Big[\\ & \omega \vec{u}_{n,j} \cdot \vec{u}_{m,i} g_A(|\vec{\rho}_{n,j} - \vec{\rho}_{m,i}|) + \frac{\beta_m \beta_n}{\omega} g_V(|\vec{\rho}_{n,j} - \vec{\rho}_{m,i}|)\Big] \\ & + \frac{1}{j\omega} e^{-j\beta_m \zeta_m} e^{-j\beta_n \zeta_n} g_V(|\vec{\rho}_{n,j} - \vec{\rho}_{m,i}|) \Big|_{\zeta_m = -\frac{L_m}{2}}^{\zeta_m = \frac{L_m}{2}} \Big|_{\zeta_n = -\frac{L_n}{2}}^{\zeta_n = \frac{L_n}{2}} \\ & - \frac{\beta_n}{\omega} \int_{-L_n/2}^{L_n/2} d\zeta_n e^{-j\beta_n \zeta_n} \Big[e^{-j\beta_m \zeta_m} g_V(|\vec{\rho}_{n,j} - \vec{\rho}_{m,i}|) \Big|_{\zeta_m = -\frac{L_m}{2}}^{\zeta_m = \frac{L_m}{2}} \\ & - \frac{\beta_m}{\omega} \int_{-L_m/2}^{L_m/2} d\zeta_m e^{-j\beta_m \zeta_m} \Big[e^{-j\beta_n \zeta_n} g_V(|\vec{\rho}_{n,j} - \vec{\rho}_{m,i}|) \Big|_{\zeta_n = -\frac{L_n}{2}}^{\zeta_n = \frac{L_n}{2}} \end{aligned}$$

This expression results in a symmetric S-parameter matrix, as it must. Utilizing the thru and reflection terms of the isolated lines, the 4-port S-parameter matrix can be completely filled.

The approximations for g_A and g_V given in [4] have been used in the following examples.

Comparison with Parallel Coupled Lines

Since the expression applies to any line orientation, it can be compared to parallel lines as a test since parallel lines are just a special case of skew lines. Figure 1 shows the test case which is that of 50Ω lines on .125 mm alumina. The center to center spacing of the lines is 4 substrate heights, so the weak coupling assumption will be valid. The frequency is held fixed, and the line length is varied from five to one hundred substrate heights. The new expression is compared to the standard model for parallel coupled lines [5] in Figure 2 thru Figure 5.

The four plots taken together demonstrate that when coupling is weakest, agreement between the standard model and our expression is worst. Since this is the case when coupling should be unimportant, this is acceptable. Overall, the plots show that phase agrees better than magnitude. The magnitude agrees within a few dB, except when the coupling approaches zero.

Skew Line Experiment

An experiment for the case of skew lines is also needed. In order to get an appropriate level of coupling, the "vee" circuit shown in Figure 6 has been fabricated and tested. Relevant parameters can be found in the figure. In order that this experiment be meaningful, the structure has been designed to push the limits of the validity of the approximations. A representative sample of the results are shown in Figure 7 and Figure 8.

The agreement between theory and experiment is much better for S_{32} than it is for S_{31}. This is because S_{31} happens to be very sensitive to small changes in the circuit. This can be seen in Figure 9 where the relative dielectric constant of the substrate has been varied by $\pm 5\%$. Nevertheless, for this extreme case and for the purpose of parasitic coupling, the agreement is considered good.

References

[1] M. Goldfarb, and A. Platzker, "The effects of electromagnetic coupling on MMIC design", **Microwave and Millimeter Wave CAE**, vol. 1, no. 1, pp. 38-47, Jan. 1991.

[2] J. Mosig, "Integral equation echniques", chapter 3, **Numerical Techniques for Microwave and Millimeter-Wave Passive Structures**, ed. by T. Itoh, New York: J. Wiley and Sons, 1989.

[3] Edward F. Kuester, Private communication.

[4] J. M. Dunn, "A uniform asymptotic expansion for the Green's functions used in microstrip calculations", **IEEE Trans. Microwave Theory Tech.**, vol. MTT-39, pp. 1223-1226, July, 1991.

[5] Reinmut K. Hoffman, **Handbook of Microwave Integrated Circuits**, Artech House, Norwood, Massachusetts, 1987.

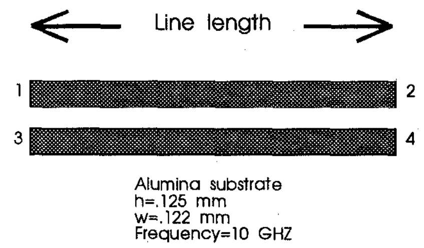

Figure 1. Port labels and dimensions for the comparison of the expression in this paper to the standard parallel coupled line model. All port impedances are $50\,\Omega$.

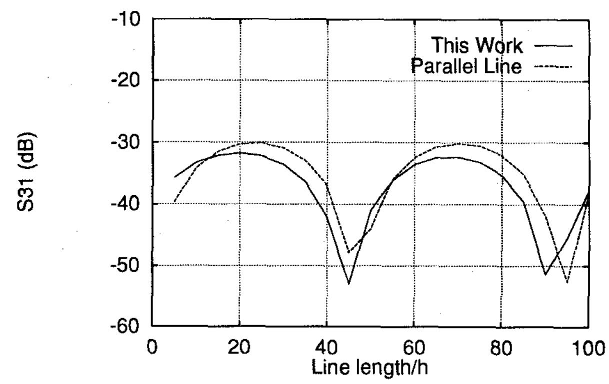

Figure 2. Coupling of lines in Figure 1.

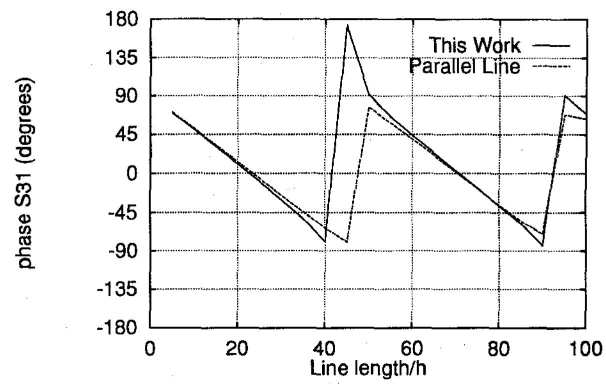

Figure 3. Coupling of lines in Figure 1.

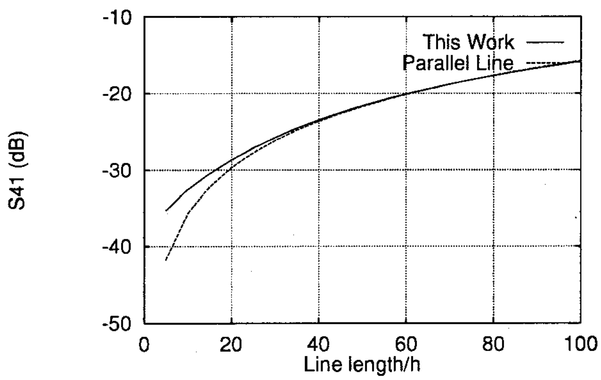

Figure 4. Coupling of lines in Figure 1.

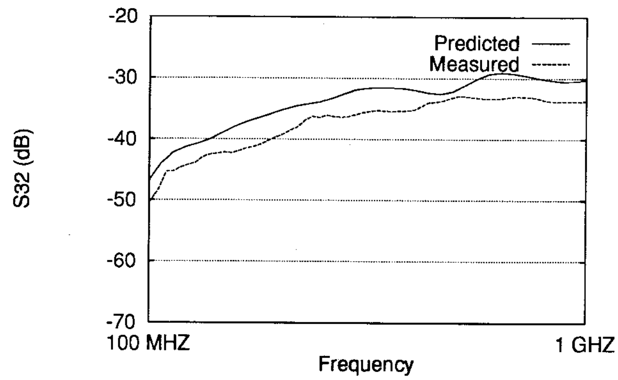

Figure 7. Measured versus calculated coupling of the *vee* circuit shown in in Figure 6.

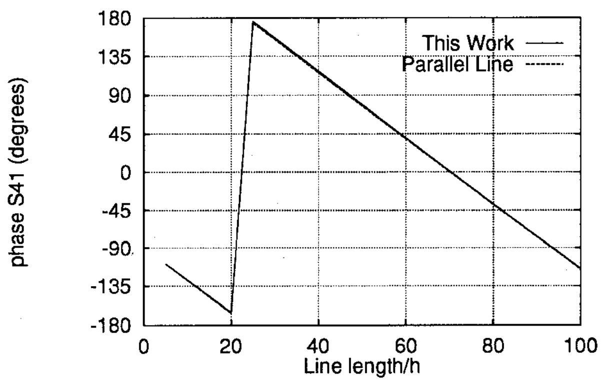

Figure 5. Coupling of lines in Figure 1.

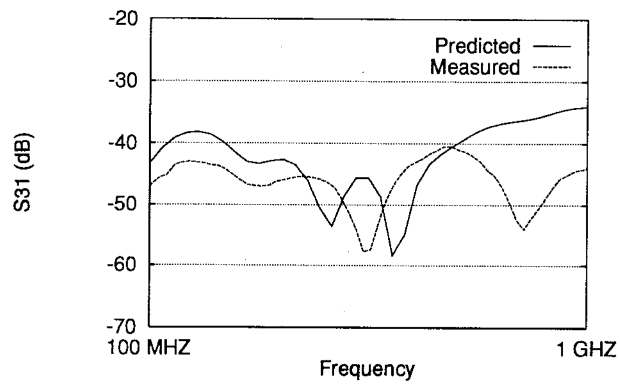

Figure 8. Measured versus calculated coupling of the *vee* circuit shown in in Figure 6.

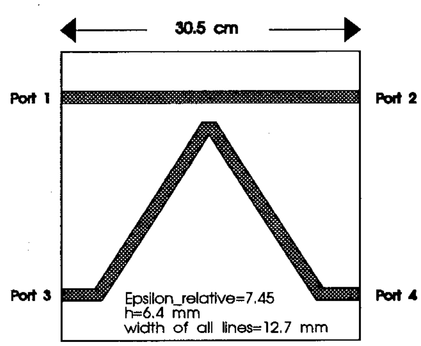

Figure 6. Port labels and dimensions of the *vee* circuit.

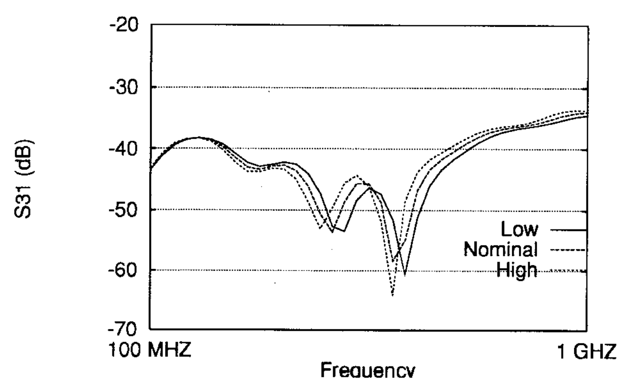

Figure 9. Calculated coupling of the *vee* circuit shown in Figure 6 for three different values of ϵ_r. Notice the large variaion in coupling for a small change in one parameter.